Determination of Metrological Structural Resolution using an Aperiodic Spatial Frequency Standard

Florian WOHLGEMUTH [1], Matthias FLEßNER [2], Tino HAUSOTTE [1]
[1] Institute of Manufacturing Metrology, Friedrich-Alexander-University Erlangen-Nürnberg, Erlangen, Germany
[2] Volume Graphics GmbH, Heidelberg, Germany

Contact e-mail: Florian.Wohlgemuth@fmt.fau.de

Abstract. The metrological structural resolution is both an important indicator of a measurement system's metrological scope (also in comparison to other measurement systems) as well as an indicator for low pass filtering that typically reduces probing errors. For computed tomography systems, it is defined in the VDI/VDE guideline 2630 Part 1.3 as 'the size of the smallest structure that can still be measured dimensionally'. The determination procedure suggested in this guideline includes the measurement of multiple calibrated microspheres which has proven expensive and time-consuming in practice. Therefore, alternative approaches to measuring the metrological structural resolution are currently being proposed and researched. Ideally, these are applicable to a wide range of measurement systems.

In prior works of the Institute of Manufacturing Metrology, an aperiodic spatial frequency standard was presented. Qualitatively, it could be shown that with this standard, the expected effects can be detected. This work presents and discusses the quantitative determination of the metrological structural resolution using this standard. The determination is based on the calculation of a spatial frequency dependent amplitude transfer function and a subsequent fit with a Gaussian function. In this work, the effect of different cutoff frequencies for the fit is studied and influence factors on the MSR values are demonstrated.

1. Introduction

The metrological structural resolution for computed tomography (CT) systems is defined in the VDI/VDE guideline 2630 Part 1.3 as 'the size of the smallest structure that can still be measured dimensionally' [1]. This abstract definition is accompanied by the suggestion to measure multiple calibrated microspheres until the measurement deviation exceeds a threshold provided by the manufacturer. The diameter of the smallest microsphere that is still measureable within the provided threshold is the structural resolution of the CT system. This determination procedure has proven unattractive in practice due to the required measurement time as well as the cost of multiple calibrated microspheres. As the metrological structural resolution can both serve as an important indicator of a measurement system's metrological scope as well as indicate low pass filtering (which typically reduces probing errors in acceptance testing [2]), an alternative approach to the determination of the metrological structural resolution is of high interest. It is advantageous if this alternative approach is also applicable to a wide range of measurement systems, as not only the capabilities of different

CT systems can be compared, but also a wide range of different sensors can be compared regarding their metrological properties.

There are projection- and volume-based resolution characterisation methods like the modulation transfer function [3, 4, 5]. From the point of view of dimensional metrology, those characterisation methods are unsatisfying because they cannot easily be transferred to all dimensional measuring systems (e.g. as other systems do not provide comparable projection or volume data). But even for CT systems, these characterisation methods cannot cover all influences relevant for dimensional metrology. The result of a dimensional measurement using a CT system is influenced by all measurement chain steps (projection acquisition, reconstruction, surface determination and subsequent dimensional evaluation (e.g. fitting of a standard geometry)). A metrologically sound resolution quantification needs to compass all measurement chain steps before the dimensional evaluation to be sensitive to all potential influence factors on the dimensional measurement result.

In the field of CT, different determination methods have been proposed for the structural resolution (e.g. [6, 7]). In prior work at the Institute of Manufacturing Metrology, an aperiodic spatial frequency standard (ASFS) was presented. Qualitatively, it could be shown that with this standard, the expected effects can be detected [8, 9]. In this contribution, we want to discuss a quantitative determination method in detail.

2. Determination Approach

In system theory, a system's transfer function $f(\vec{x})$ describes its properties by connecting system input $i(\vec{x})$ with the system output $o(\vec{x})$:

$$o(\vec{x}) = \int f(\vec{x} - \vec{y})\, i(\vec{y}) d\vec{y} \tag{1}$$

This convolution can be rewritten as a multiplication in Fourier space, yielding a simple determination formula for the system's transfer function:

$$\tilde{f}(\vec{k}) = \frac{\tilde{o}(\vec{k})}{\tilde{\imath}(\vec{k})} \tag{2}$$

The basic idea behind the metrological structural resolution determination using the aperiodic spatial frequency standard is to characterise a measurement system by its transfer function. The aperiodic spatial frequency standard (see Fig. 1) is a rectangular cuboid of dimension 4 mm x 4 mm x 26 mm. It is made of titanium and manufactured using laser ablation with one of the longer side faces featuring aperiodic structures. The structures are constant along the shorter 4 mm direction of the side face and vary without a reproducible pattern along the longer 26 mm side. The total length of the aperiodic structure is 25 mm, leaving 0.5 mm of even surface on each end of the standard that are useful e.g. for tactile profilometer measurements for initial positioning of the stylus. For the determination of the metrological structural resolution, the CT system's transfer function is determined for profile lines extracted along the standard (compare section 3), yielding a 'one dimensional metrological structural resolution'.

The aperiodic spatial frequency standard is designed to have amplitudes over a broad frequency range in order to effectively sample a good range of the transfer function. Typical structure sizes are in the order of 100 µm which is also the order of structural resolutions that can be determined using this standard. The maximum inclination is 45° to allow for a wide range of optical measurements. The design spectrum of the aperiodic spatial frequency standard (based on a profile line taken from the CAD) is shown in Fig. 2.

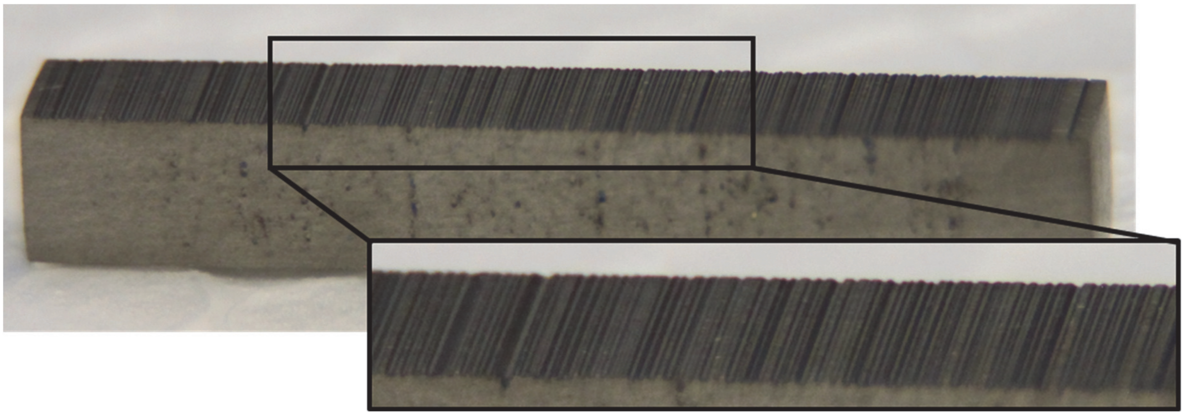

Fig. 1. Photograph of the aperiodic spatial frequency standard.

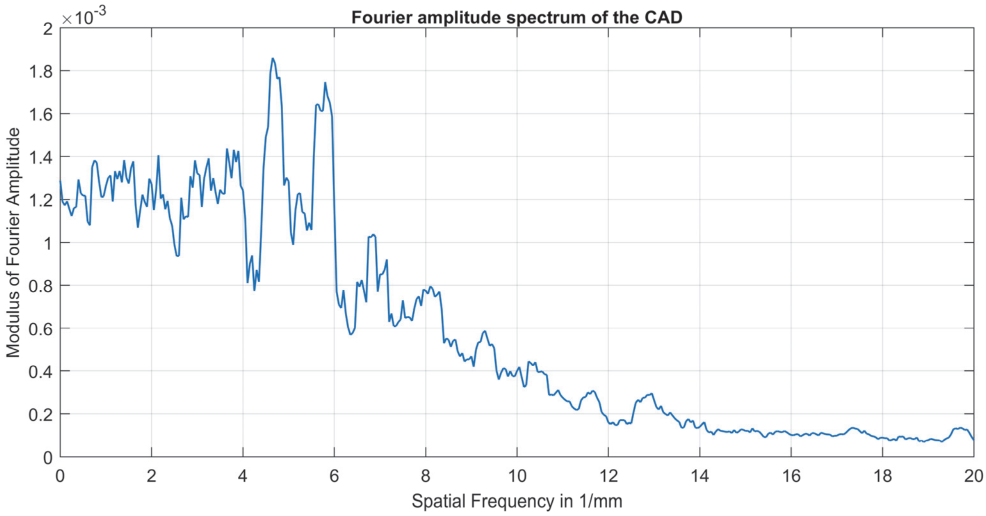

Fig. 2. Aperiodic spatial frequency standard CAD spectrum.

3. Methodology

For the determination procedure using the aperiodic spatial frequency standard, the same profile is measured both with a reference measurement system of nominally significantly higher resolution as well as with the measurement system to be characterised. For the case of CT, a tactile profilometer can serve as reference measurement system. Alternatively, if simulations using a CT digital twin are used, the simulation mesh can serve as reference data set. The main idea is to perform a Fourier transform of reference and measurement profile and then determine the transfer function by analysing the ratio of the measurement and the reference profile amplitudes. In the following, the single data processing steps are described in detail. Fig. 3 shows a conceptual summary.

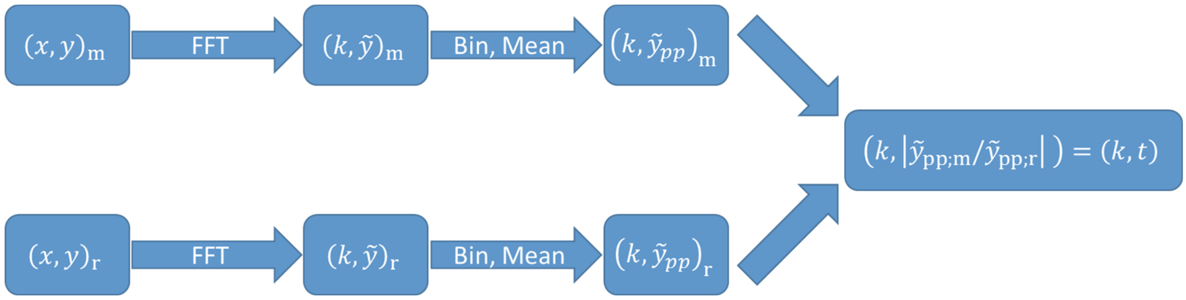

Fig. 3. Determination of the transfer function (k, t) (k is the spatial frequency variable, t the amplitude transfer at that frequency) from the (aligned) measured profile values $(x, y)_m$ and a reference profile $(x, y)_r$.

3.1 Alignment

The measurement data both for reference measurement system and measurement system is a 2D data set of a profile. Due to the working principle of some measurement systems, a reasonable alignment of the profile direction and the coordinate system the profile was measured in is not always guaranteed. To solve this, the measurement data (x, y) is fitted with a robust (bisquare weighted [10]) linear fit $y = p_1 \cdot x + p_2$ in MathWorks Matlab 2019a. The constant offset is then subtracted from each data point ($y \mapsto y - p_2$) and subsequently, the data set is rotated with an angle of $-\arctan(p_1)$ to ensure that the x coordinate represents the coordinate along the profile and y the profile height at that x value. To ensure comparability, this alignment procedure is performed for data sets from any measurement system.

3.2 Fourier Transform

The measurement and reference measurement data after alignment result in a profile of the measurement $\{(x_{1;\text{measured}}, y_{1;\text{measured}}), \dots, (x_{n;\text{measured}}, y_{n;\text{measured}})\}$ as well as a reference profile $\{(x_{1;\text{reference}}, y_{1;\text{reference}}), \dots, (x_{n;\text{reference}}, y_{n;\text{reference}})\}$. In this study, a constant profile length of 25.2 mm is used for the CT measurement data. In general, a comparable profile length for different measurement systems should be chosen to guarantee comparability of the profiles and spectra obtained. The y-values of both profiles are then transformed to the spatial frequency domain. The result are two complex valued amplitude vectors $\{\tilde{y}_{1;\text{measured}}, \dots, \tilde{y}_{n;\text{measured}}\}$ and $\{\tilde{y}_{1;\text{reference}}, \dots, \tilde{y}_{n;\text{reference}}\}$ as well as two corresponding spatial frequency vectors $\{k_{1;\text{measured}}, \dots, k_{n;\text{measured}}\}$ and $\{k_{1;\text{reference}}, \dots, k_{n;\text{reference}}\}$. In the following, only the complex amplitude (and not the phase) of the FFT is of interest. Thus, the spectral data contains redundant information and it suffices to consider the positive spatial frequency part with indices $\{1, \dots, m\}$.

3.3 Binning and Filtering

Due to unavoidable differences in the sampling positions $\{x_{1;\text{measured}}, \dots, x_{n;\text{measured}}\}$ and $\{x_{1;\text{reference}}, \dots, x_{n;\text{reference}}\}$ (at least due to limited numerical precision, but also due to different sampling of different measurement devices), the measurement and reference spatial frequency vectors $\{k_{1;\text{measured}}, \dots, k_{m;\text{measured}}\}$ and $\{k_{1;\text{reference}}, \dots, k_{m;\text{reference}}\}$ are not identical. To obtain a transfer function from the amplitude spectra $\{|\tilde{y}_{1;\text{measured}}|, \dots, |\tilde{y}_{m;\text{measured}}|\}$ and $\{|\tilde{y}_{1;\text{reference}}|, \dots, |\tilde{y}_{m;\text{reference}}|\}$, their ratio has to be calculated at the same spatial frequencies (compare equation (2) on page 2). To obtain their values at identical spatial frequencies, the amplitude spectra are rebinned with a fixed spatial frequency bin width Δ_{bin}. In this study, 0.05 mm^{-1} was used as bin width. The binning process yields two binned amplitude spectra $\{|\tilde{y}_{1;\mathrm{measured;b}}|, \dots, |\tilde{y}_{h;\mathrm{measured;b}}|\}$ and $\{|\tilde{y}_{1;\mathrm{reference;b}}|, \dots, |\tilde{y}_{h;\mathrm{reference;b}}|\}$ with one corresponding frequency vector $\{0, \dots, (h-1) \cdot \Delta_{\mathrm{bin}}\}$. In principle, a transfer function can already be calculated from this data. In practice, there is a lot of noise on these binned amplitude spectra. To suppress that, a moving mean filter is used to smoothen the spectra. For this study, a filter width of 9 bins was used. After the application of this filter, the result of this data processing are two binned and filtered, post-processed amplitude spectra - $\{|\tilde{y}_{1;\mathrm{m;pp}}|, \dots, |\tilde{y}_{h;\mathrm{m;pp}}|\}$ for the measurement and $\{|\tilde{y}_{1;\mathrm{r;pp}}|, \dots, |\tilde{y}_{h;\mathrm{r;pp}}|\}$ for the reference.

3.4 *Transfer Function and Fit*

The transfer function at the spatial frequencies $\{0, \dots, (h-1) \cdot \Delta_{\mathrm{bin}}\}$ is calculated as $\{|\tilde{y}_{1;\mathrm{m;pp}}|/|\tilde{y}_{1;\mathrm{r;pp}}|, \dots, |\tilde{y}_{h;\mathrm{m;pp}}|/|\tilde{y}_{h;\mathrm{r;pp}}|\}$. This transfer function now serves to characterise the system response for the spatial frequency range. Empirically, a Gaussian characteristic of this transfer function could be observed for the CT systems at the Institute of Manufacturing Metrology. Thus, the transfer function is fitted with a Gaussian function of the form:

$$f(k) = \exp\left(-\ln(2) \cdot \frac{k^2}{k_{\mathrm{c}}^2}\right) \tag{3}$$

The transfer function has a limit of 0 for $k \to \infty$, signifying that arbitrarily small structures cannot be resolved. Additionally, the transfer function is set to be 1 (signifying an amplitude transfer of 100 %) for $k = 0$ by definition.

To fit the data points at all spatial frequencies $\{0, \dots, (h-1) \cdot \Delta_{\mathrm{bin}}\}$ is usually not constructive as the nominal spectrum's information content (i.e., its amplitudes) drop drastically at a certain frequency (compare Fig. 2), leading to high noise levels due to the division by a small reference measurement amplitude. Therefore, it is useful to set a cutoff frequency for the fit. We discuss the influence of this cutoff frequency and its choice in Section 4.4. Our usual choice for the cutoff frequency in this study is 13 mm^{-1}.

k_{c} is the spatial frequency for which the transmission characteristic of the system drops to 50 %. The metrological structural resolution l_{MSR} is calculated as the corresponding length:

$$l_{\mathrm{MSR}} = \frac{1}{k_{\mathrm{c}}} \tag{4}$$

The choice of the 50 % transmission criterion is based on the analogy to other filters used in dimensional metrology [11] and agrees with current standardisation in other dimensional metrology fields [12]. Future research might lead to the conclusion that p% transmission are a more suitable measure to represent a dimensional metrological system's capabilities. In this case, the new value for the structural resolution would be given by:

$$l_{\mathrm{MSR}} \cdot \sqrt{\frac{\ln\left(\frac{p}{100}\right)}{\ln\left(\frac{1}{2}\right)}} \tag{5}$$

4. Results

4.1 Setup

Reference measurements were carried out using both a tactile profilometer (Taylor Hobson (Leicester, England) Form TalySurf Serie 2 PGI, 2 µm diamond tip stylus) and an optical focus variation system (Alicona (Raaba/Graz, Austria) Infinite Focus G4 using a 50 x objective). The profilometer measurement was repeated four times, the Alicona measured a surface of 26.45 mm x 0.24 mm. Due to the working principle of the Alicona, inclinations can only be measured up to a certain acceptance angle. This causes gaps in the measured surface that have no height information. As 332 parallel profiles can be extracted from the surface data, the profile with the fewest gap points was chosen for further use as reference profile. For this profile, the ratio of gaps is 1.7 %. The missing data points from this profile were interpolated linearly for the Fourier transform.
The titanium ASFS (compare Section 2 and Fig. 1) was measured using a Carl Zeiss Industrielle Messtechnik GmbH (Oberkochen, Germany) Metrotom 1500 225kV (2048 x 2048 pixel detector with 200 µm pixel size). Measurements were performed using the parameters given in Table 1. Measurement M1 is a measurement with a small nominal spot size to minimise spot blurring. M2 represents a typical choice for a good measurement while measurement M3 is a measurement with high unsharpness and a shorter measurement time due to a large focal spot (high tube power).

Table 1. Measurement parameters

Measurement Number	**M1**	**M2**	**M3**
	small spot	normal spot	large spot
Voltage/kV	200	200	200
Current/µA	91	182	546
Filter	0.5 mm Cu	0.5 mm Cu	0.5 mm Cu
Magnification	6.54	6.54	6.54
Voxel Size/µm	30.56	30.57	30.56
Nominal Spot Size/µm	18	36	109
Number of Projections	1600	1600	1600
Binning	1 x 1	1 x 1	1 x 1
Integration Time/ms	2000	2000	667
Detector Gain	16	16	16
Image Averaging	none	none	none

The obtained volume data from the CT measurements was furtheron processed using VGSTUDIO MAX 3.2.5. The surface was determined using the local adaptive surface determination with a search radius of 4 voxels from the iso50 surface (iso50 value determined automatically by VGSTUDIO MAX). It was then registered onto the CAD using a Best Fit registration and a measurement template consisting of 11 parallel line fits along the standard (compare Fig. 4) was applied. These lines are spread with a spacing of 0.3 mm and an edge distance of 0.5 mm. To ensure that no points on the surface are excluded from the fit, a search distance of 0.2 mm and a maximum gradient of 90° were used as fitting parameters. The fit points of those lines were exported as csv-files and processed in MATLAB as described in section 3 to obtain values for the metrological structural resolution. The intent behind using 11 line fits was to detect noise. For the purpose of this paper, we evaluated only one line per measurement and we always evaluated the same line on the standard for all measurements.

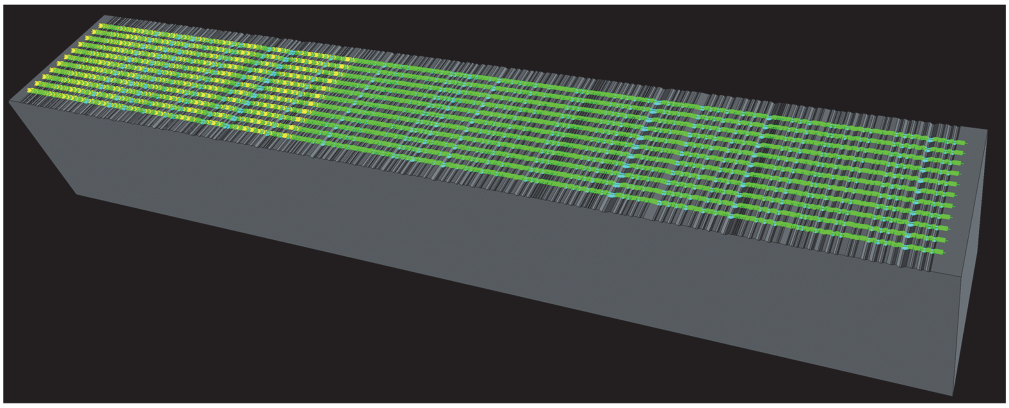

Fig. 4. Measurement template defined on the CAD file in VGSTUDIO MAX showing the 11 parallel lines for the profile extraction from the standard.

4.2 Influence of the Focal Spot Size

Using the same profilometer measurement as reference, the metrological structural resolution (MSR) for measurement M1 is 117 µm, the MSR value for M2 is 151 µm and the MSR for measurement M3 is 209 µm. The transmission spectrum and the Gaussian fit for measurement M2 are shown in Fig. 5. The MSR determination can thus clearly register the loss of information due to the increased focal spot smearing.

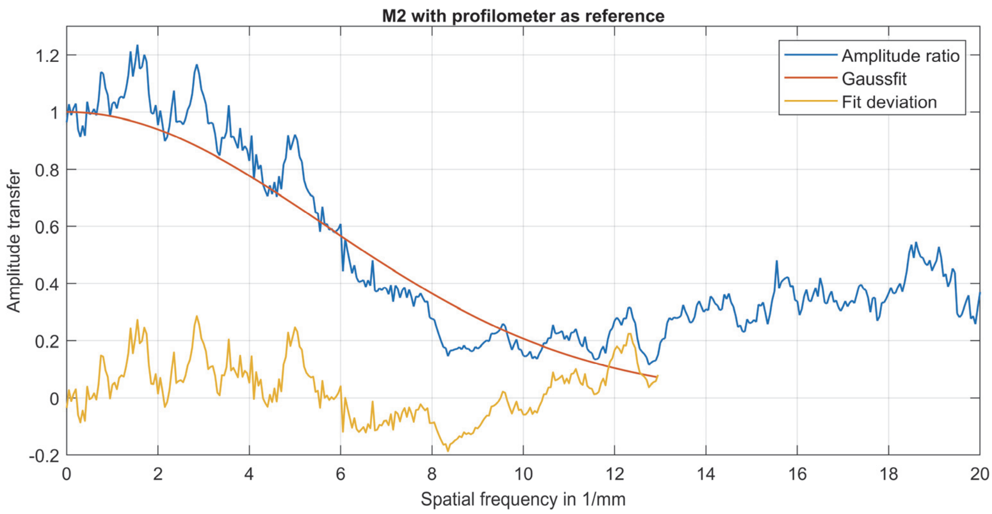

Fig. 5. Transmission spectrum and Gaussian fit with fit deviation for measurement M1.

4.3 Choice of Reference Measurements

The four profilometer measurements show a small variation concerning their amplitude spectra – especially when compared to the difference between a profilometer measurement spectrum and a focus variation measurement spectrum (compare Fig. 6 and Fig. 7). The differences are likely to be caused by the different transmission spectra that characterise the profilometer and the focus variation, respectively. Due to the lack of a higher resolution reference measurement, no spectrum can be regarded as inherently 'better' or 'more true' than the other.

The MSR values determined using all five reference spectra (four profilometer spectra, one focus variation spectrum) are in the range of 113 µm to 118 µm, 149 µm to 153 µm and 207 µm to 215 µm for measurements M1, M2 and M3, respectively. The span of those values (5 µm, 4 µm, 8 µm) can be seen as a minimum value for the reference measurement's contribution to the uncertainty of the MSR determination. We however believe that there are other influence factors with significantly higher contributions to the uncertainty of the determined MSR value and that thus both profilometer and focus variation measurements can be seen as valid reference measurements.

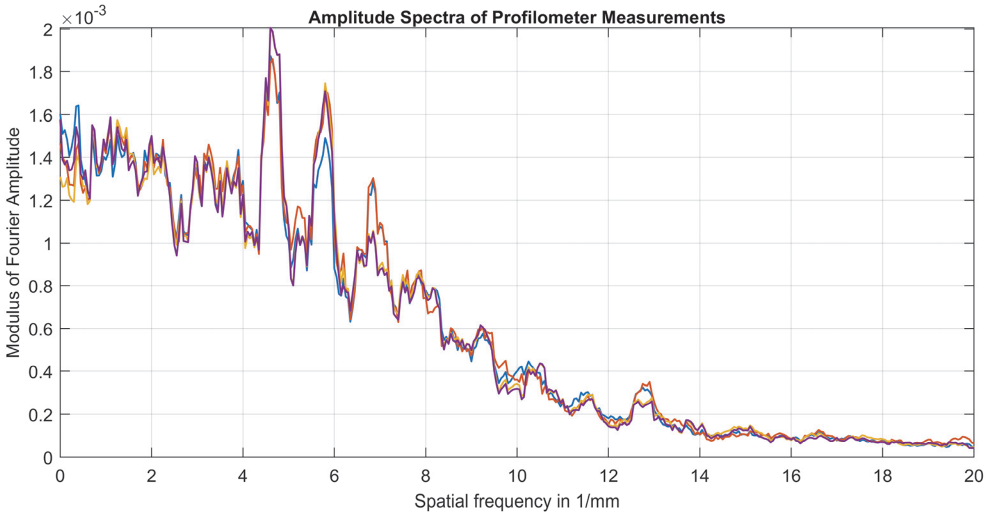

Fig. 6. Fourier amplitude spectra of the four different profilometer reference measurements.

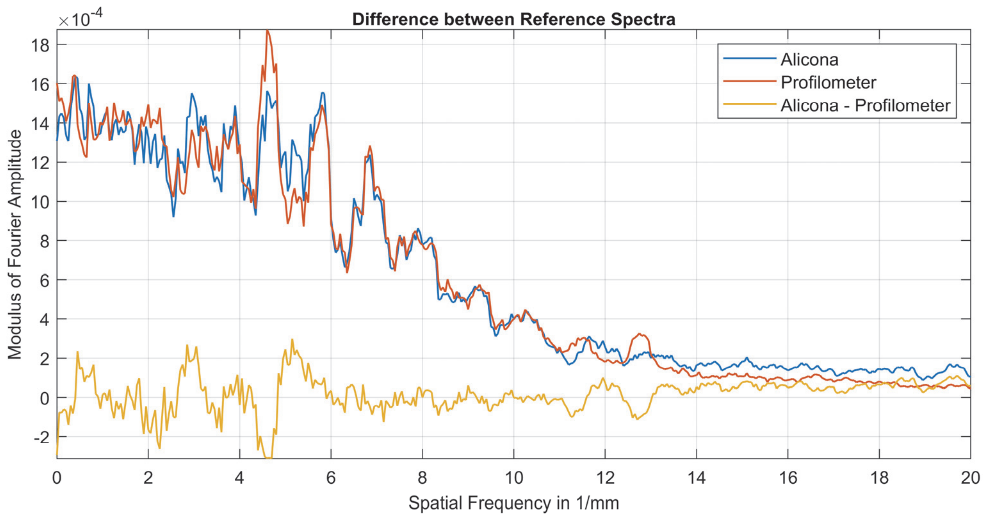

Fig. 7. Fourier amplitude spectra of a profilometer reference measurement and the focus variation reference measurement along with the amplitude difference.

4.4 Influence of the Fit Cutoff Frequency

As already mentioned in Section 3.4, the Gaussian fit uses only spectrum amplitudes up to a certain maximum spatial frequency to minimise the impact of high frequency noise. The choice of input data for the fit also influences the fit result and thus the determined MSR value. Therefore, the choice of the fit cutoff frequency (13 mm^{-1} in this work) is important. We chose this value due to the nominal frequency spectrum (Fig. 2) and the reference measurement spectra (Fig. 6 and Fig. 7). All these spectra have a last amplitude peak shortly before 13 mm^{-1} and drop to lower amplitudes at higher frequencies. As we think that the division by small reference amplitudes is the main reason for non-negligible high frequency 'noise' amplitudes distorting the measurement, basing the choice on this criterion seems reasonable and should guarantee comparability. We believe that the fit cutoff frequency needs to be a design parameter set for each specific determination standard.

We further believe that the cutoff frequency value should be significantly larger than the critical frequency k_c (compare equation (3) and (4)) as otherwise, the MSR value is determined from an extrapolation. A cutoff frequency of 13 mm^{-1} corresponds to 77 µm which is thus the minimal MSR that could be determined.

Fig. 8 shows the dependence of the determined MSR value on the cutoff frequency used. In particular, there are pronounced differences for frequencies above ~15 mm^{-1}. In this spatial frequency region, the Alicona reference spectrum has significantly higher reference amplitudes (compare Fig. 7). Thus, the transfer spectrum values are significantly lower, not shifting the Gaussian fit function to higher spatial frequencies (and thus, lower MSR values). This again indicates that our idea of small reference amplitudes being problematic should hold. Between 8 mm^{-1} and 15 mm^{-1}, the MSR values determined with both reference measurements agree to 2 µm.

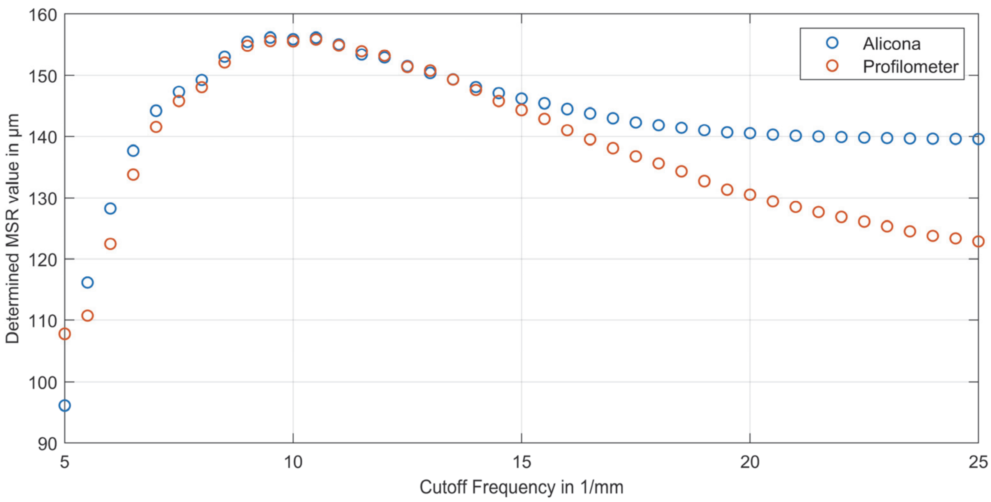

Fig. 8. Determined MSR value for measurement M1 and both reference measurements depending on the cutoff frequency used.

4.5 Influence of Meshing

For many metrology applications, the determined surface is converted to a mesh (e.g. in STL format) prior to subsequent processing. It is thus of interest to inspect the influence of this meshing on the structural resolution. For this purpose, the measurement volume of M1 was imported into VGSTUDIO MAX and subsequent to surface determination, the surface was converted to three different meshes using the presets 'Ray-based fast', 'Ray-based normal' and 'Ray-based precise'. Then, the MSR was determined for the volume surface as well as for the mesh surfaces. The results are summarised in Table 2. As reference, the first profilometer measurement was used.

For measurement M1, the preset 'Ray-based fast' results in a similar loss of metrological resolution as a spot of double size (compare Section 4.2). The 'Ray-based normal' and 'Ray-based precise' presets result in small differences.

Table 2. MSR values for different meshing

Measurement on:	**Volume surface**	**Mesh with Preset Ray-based fast**	**Mesh with Preset Ray-based normal**	**Mesh with Preset Ray-based precise**
M1:MSR value in µm	117	152	119	120

5. Conclusion

The quantitative determination of metrological structural resolution values using the aperiodic spatial frequency standard (ASFS) was described in detail. The determination comprehends the calculation of a Fourier spectrum, the determination of a transfer spectrum and a Gaussian fit. It could be shown that the determination method using this standard is able to detect changes in resolution due to spot size changes and is further able to detect loss of resolution due to coarse STL-meshing. The choice of the right reference measurement and a suitable cutoff frequency for the Gaussian fit was discussed.

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